Technical Notes

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Kinetically Controlled Vaporization of a Polyatomic Gas

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Introduction

WHEN large interphase mass-transfer rates occur, as with the sublimation of a solid or the vaporization of a liquid, a thin nonequilibrium region occurs at the phase interface. Such a region, a few tens of mean-free paths in thickness and usually referred to as the Knudsen layer, is shown schematically in Fig. 1. At the interface, molecules leave the condensed phase with a half-Maxwellian velocity distribution associated with the surface temperature. At the Knudsen layer edge, a continuum velocity distribution is established at a lower temperature. The objective of a Knudsen layer analysis is to predict the "jump conditions" across the layer.

Early analyses of the problem,[1-5] including that of Knudsen,[1] were based on linearized equations. More recently, nonlinear analyses have been reported.[6-9] Anisimov[6] considered the case of a monatomic gas vaporizing into a vacuum. Poyurovskaya[7] extended the Ref. 6 results to include kinetically controlled vaporization of a monatomic gas and presented a series expansion solution. Knight[8] and Cercignani[9] independently extended Anisimov's analysis to include internal degrees of freedom of the vaporizing molecules, i.e., a polyatomic gas. Their analytic results, however, did not include kineticaly controlled vaporization effects.

The purpose of this note is to present recently developed analytic solutions to a nonlinear analysis of the Knudsen layer, which includes the effects of both a polyatomic gas and vaporization kinetics.

Problem Formulation

Referring to Fig. 1, a Knudsen layer model provides a link between measurable macroscopic variables at the Knudsen layer edge and the equilibrium thermodynamic properties of the saturated vapor at the solid (or liquid) surface. Following the approach of Anisimov,[6] as extended by Knight[8] to include internal degrees of freedom of the subliming/vaporizing molecules, the conservation equations of mass, momentum, and energy for the Knudsen layer are

$$\rho u = \alpha\rho_s\sqrt{\frac{RT_s}{2\pi}} + \alpha\beta\rho\sqrt{\frac{RT}{2\pi}}\left[\pi^{1/2}\,\mathrm{merfc}(m) - e^{-m^2}\right] \tag{1}$$

$$\rho(u^2+RT) = \frac{1}{2}\alpha\rho_s RT_s + (2-\alpha)\beta\rho RT \times\left[\left(m^2+\frac{1}{2}\right)\mathrm{erfc}(m) - \frac{m}{\pi^{1/2}}e^{-m^2}\right] \tag{2}$$

$$\rho u\left(\frac{5}{2}RT+\frac{1}{2}u^2\right) = \alpha\rho_s\sqrt{\frac{RT_s}{2\pi}}\left[2RT_s + G(\gamma)R(T_s-T)\right] + \alpha\beta\rho RT\sqrt{\frac{RT}{2\pi}}\left[m\left(m^2+\frac{5}{2}\right)\pi^{1/2}\,\mathrm{erfc}(m) - (m^2+2)e^{-m^2}\right] \tag{3}$$

All terms in these equations are defined below. The terms containing the vaporization coefficient α are treated in the same manner as in Refs. 4 and 7. It has been implicitly assumed that the true vaporization coefficient α_v and the condensation coefficient α_c are equal, i.e., $\alpha=\alpha_v=\alpha_c$. The macroscopic flow variables are the density ρ, the velocity u, and the temperature T. Properties at the edge of the Knudsen layer are unsubscripted whereas those at the surface or saturation conditions have a subscript s. The ratio of specific heats γ enters through the function $G(\gamma)=(5-3\gamma)/[2(\gamma-1)]$; thus, $2G$ is the number of internal degrees of freedom.[10] The speed ratio m is given by $m=u/\sqrt{2RT}$, and R is the universal gas constant. The velocity distribution functions assumed in obtaining Eqs. (1-3) are the same as those of Refs. (6-8).

Analytic Solutions

Equations (1-3) may be algebraically manipulated to obtain the following analytical expressions for T/T_s, ρ/ρ_s, and β as functions of m, γ, and α

$$\frac{T}{T_s} = \left[\frac{-\pi^{1/2}mF(\gamma)\varphi(m,\alpha)+\sqrt{\pi\left[mF(\gamma)\varphi(m,\alpha)\right]^2+4\left\{1+\left[1-\varphi(m,\alpha)\right]\left[2m^2+1\right]F(\gamma)\right\}}}{2\left\{1+\left[1-\varphi(m,\alpha)\right]\left[2m^2+1\right]F(\gamma)\right\}}\right]^2 \tag{4}$$

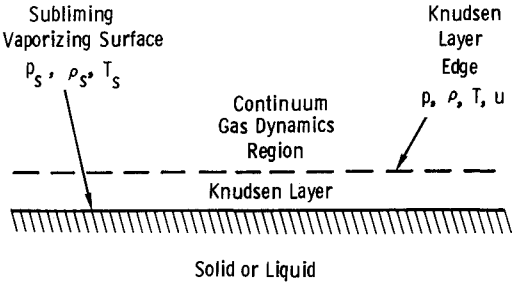

Fig. 1 Knudsen layer.

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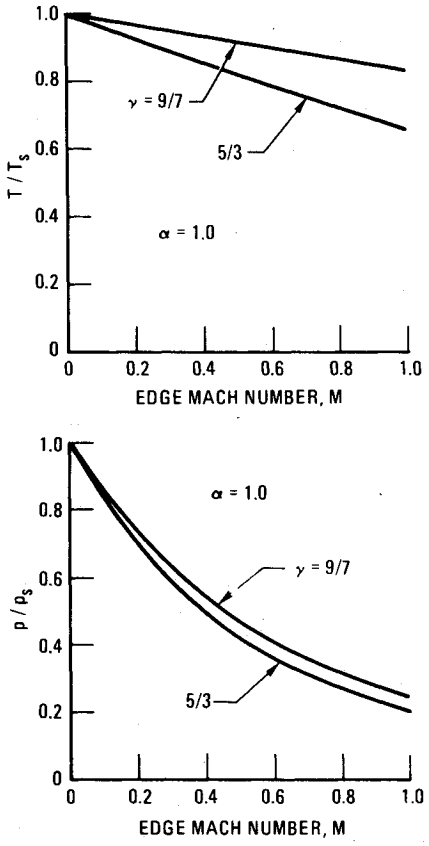

Fig. 2 Effect of γ on T/T_s and p/p_s for $\alpha = 1$.

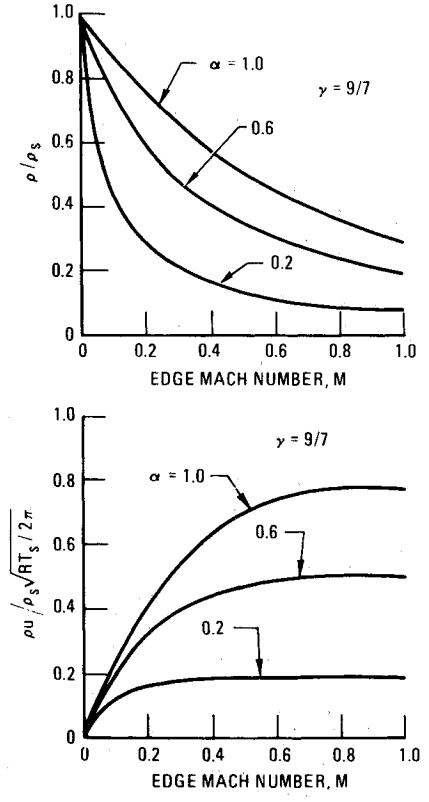

Fig. 3 Effect of vaporization coefficient α on ρ/ρ_s and mass flux ratio for $\gamma = 9/7$.

$$\frac{\rho}{\rho_s} = \varphi(m,\alpha)\left\{(2-\alpha)\left[\left(m^2+\frac{1}{2}\right)\mathrm{erfc}(m)e^{m^2} - m\pi^{-1/2}\right]\sqrt{\frac{T_s}{T}} + \frac{\alpha}{2}\left[1-\pi^{1/2}\,\mathrm{merfc}(m)\,e^{m^2}\right]\frac{T_s}{T}\right\} \tag{5}$$

$$\beta = \varphi(m,\alpha)\left[(2m^2+1) - m\pi^{1/2}\sqrt{\frac{T_s}{T}}\right]e^{m^2}\frac{\rho_s}{\rho}\sqrt{\frac{T_s}{T}} \tag{6}$$

where

$$F(\gamma) = (\gamma-1)/(\gamma+1)$$

and

$$\varphi(m,\alpha) = \left\{1 - \frac{4(1-\alpha)}{\alpha}\left[m^2 - \pi^{1/2}m\left(m^2+\frac{1}{2}\right)\mathrm{erfc}(m)e^{m^2}\right]\right\}^{-1}$$

For $\alpha = 1$, $\varphi(m,\alpha)$ is unity, and Eqs. (4-6) reduce to those of Knight.[8] In the following section, the solution behavior is illustrated.

Calculated Results

Equations (4) and (5) may be solved to obtain T/T_s and ρ/ρ_s for arbitrary values of the vaporization coefficient α and the ratio of specific heats γ. The independent variable is usually taken to be the Mach number at the edge of the Knudsen layer $M = (\sqrt{2/\gamma})m$. The pressure ratio p/p_s and the mass flux ratio $\rho u/\left[\rho_s\sqrt{RT_s/2\pi}\right]$ are obtained from

$$\frac{p}{p_s} = \frac{\rho}{\rho_s}\frac{T}{T_s} \tag{7}$$

$$\frac{\dot{m}}{\dot{m}_s} = \frac{\rho u}{\rho_s\sqrt{\dfrac{RT_s}{2\pi}}} = 2\pi^{1/2}m\frac{\rho}{\rho_s}\sqrt{\frac{T}{T_s}} \tag{8}$$

The effect of the ratio of specific heats γ on the temperature and pressure ratios across the Knudsen layer for a unit value of the vaporization coefficient α is shown in Fig. 2. For four internal degrees of freedom ($\gamma = 9/7$), the temperature ratio at the sonic point, $M = 1$, is 27% higher than for $\gamma = 5/3$, and the pressure ratio is about 19% higher. Since temperature ratio and pressure ratio both increase with increasing γ, the effect of a larger γ on the density ratio is much less, i.e., about a 7% increase for $\gamma = 9/7$ at the sonic point. The mass flux ratio decreases slightly when internal degrees of freedom are added. The decrease for $\gamma = 9/7$ in comparison to $\gamma = 5/3$ is about 10% for an edge Mach number of 0.4 and approximately 6% at the sonic point.

The effect of nonunit values of the vaporization coefficient ($\alpha = 0.6$ and 0.2) on the density and mass flux ratios for $\gamma = 9/7$ is shown in Fig. 3. At the sonic point, $M = 1$, the decrease in these ratios is very nearly proportional to $(1-\alpha)$, i.e., for $\alpha = 0.2$, the decrease is about 78%. The effect of decreasing α on the pressure ratio is very similar. However, the effect on the temperature ratio is minimal. Calculated results for $\gamma = 5/3$ and α values ranging from unity down to 0.02 showed excellent agreement (maximum error about 1%) with results calculated from the series expansion solutions of Poyurovskaya.[7]

Summary

A nonlinear analysis of the nonequilibrium region occurring at a phase change interface, i.e., a Knudsen layer, has been carried out, including both the effects of a polyatomic gas and a rate-controlled vaporization process. Analytic solutions have been obtained for the temperature, pressure, density, and mass flux ratios across the layer as a function of Knudsen layer edge Mach number M, the ratio of specific heats γ, and the vaporization coefficient α. Representative solution behavior has been shown for arbitrarily chosen γ and α. For special cases, the present results are in agreement with prevously published work.

Acknowledgments

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Calculation of Merging Turbulent Wakes

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Introduction

THE work described here can be regarded as an extension of that of Cebeci et al.[1] and Chang et al.,[2] who performed calculations for wall boundary layers and wakes, respectively, and assessed the relative merits of eddy-viscosity hypotheses based on the algebraic formulation of Cebeci and Smith[3] and on the two-equation model of Hanjalic and Launder.[4] These investigations showed that the results obtained by the two turbulence models were similar and agreed well with experiments.

Flows considered in this Note represent a further step towards the flows of aircraft; on this occasion the merging of multiple wakes downstream of a high-lift arrangement of flaps. They involve a series of scales characteristic of the neighboring wakes and jets and present challenge for calculation methods. As described in the following section, the two-equation model can be used in the same form as for the earlier work,[4] but the algebraic expression requires modification to deal with the interactive region between wakes. The results allow consideration of the advantages of the two approaches, one with added algebraic complications but cheap and easy to formulate and use, and the other requiring no change to the turbulence model but more expensive and less convenient to use.

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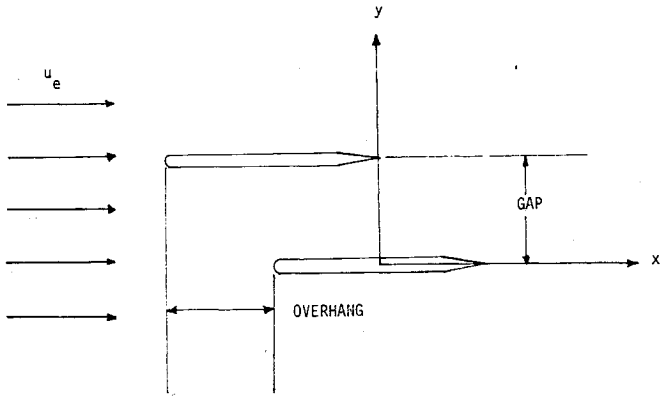
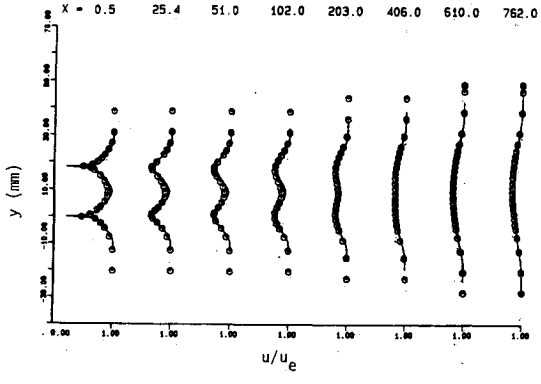


Fig. 1 System configuration and coordinate.



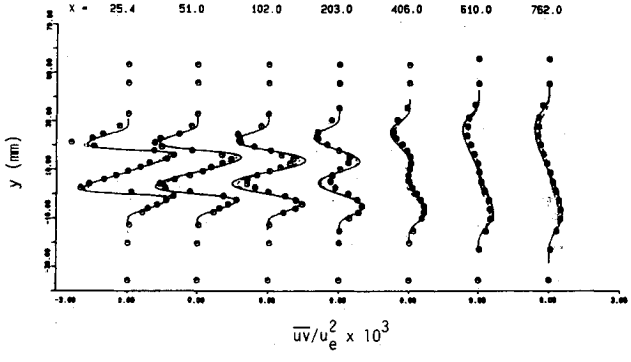


Fig. 2 Velocity profiles for overhang of 0.0 mm and gap of 19.0 mm.

Equations

The flow configuration is shown on Fig. 1 together with the notation of the following equations and results. It corresponds to the merging of wakes with different origins and has been examined experimentally by Nakayama et al.[5]

The boundary-layer form of equations for momentum, turbulence energy, and dissipation rate were solved with Keller's box scheme[6] for specified freestream velocities or boundary conditions and with the measured streamwise velocity profile at the trailing edge of the flat plate.The static pressure was assumed constant, and the initial profile of dissipation rate was obtained from the relationship in which the production and dissipation rates are the same. Edge boundary conditions for kinetic energy and dissipation rate corresponded to reduced forms of the conservation equations with zero cross-stream gradients and known values of external velocity u_e.

The algebraic eddy viscosity expression for the wake is that of Chang et al.[2] which, with B_1 given by $(x-x_{te})/20\delta_{te}$, is

$$\epsilon_m = \epsilon_w + (\epsilon_{te} - \epsilon_w)\bar{e}^{B_1} \tag{1}$$